1-2 Linear Measure

Find the length of each line segment or object.

1. Refer to Page 18.

SOLUTION:

The ruler is marked in centimeters. The tail of the fish starts at the zero mark of the ruler and the mouth appears to end 7 tenth marks after 5. So, the length of the fish is about 5.7 cm or 57 mm.

2. 

SOLUTION:

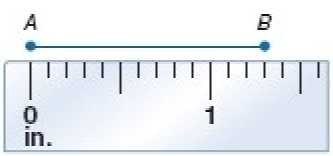
The ruler is marked in centimeters. The starting point *C* of the line segment is at the zero mark of the ruler and the other end point *D* is 5 tenth marks after 3. So, the length of the line segment is 3.5 cm or 35 mm.

3. Refer to Page 18.

SOLUTION:

The ruler is marked in inches. The distance between two consecutive numbers is divided into 16 equal parts. One end of the butterfly starts at the zero mark of the ruler and the other end ends 14 marks after 1. So, the length of the butterfly is

$1\frac{14}{16}$ in. or $1\frac{7}{8}$ in..

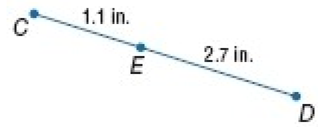
4. 

SOLUTION:

The ruler is marked in inches. The distance between two consecutive numbers is divided into 10 equal parts. The starting point *A* of the line segment is at the zero mark of the ruler and the other end point *B* is 3 tenth marks after 1. So, the length of the line segment is $1\frac{3}{10}$ in.

Find the measurement of each segment. Assume that each figure is not drawn to scale.

5. $\overline{CD}$



SOLUTION:

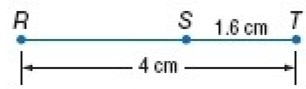
$CD = CE + ED$ Betweenness of points

$CD = 1.1 + 2.7$ Substitution.

$CD = 3.8$ Add.

So, *CD* = 3.8 in.

6. $\overline{RS}$



SOLUTION:

$RT = RS + ST$ Betweenness of points

$RT - ST = RS + ST - ST$ $-ST$ from each side.

$4 - 1.6 = RS$ Substitution.

$2.4 = RS$ Subtraction.

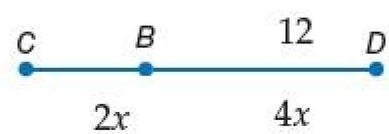
So, *RS* = 2.4 cm.

ALGEBRA Find the value of *x* and *BC* if *B* is between *C* and *D*.

7. *CB* = 2*x*, *BD* = 4*x*, and *BD* = 12

SOLUTION:

Here *B* is between *C* and *D*.



$BD = 4x$ Given

$12 = 4x$ Replace BC with 12.

$\frac{12}{4} = \frac{4x}{4}$ Divide each side by 4.

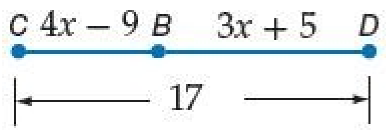
$3 = x$ Simplify.

Therefore, *x* is 3. To find *BC*. find 2*x* or 2(3) = 6. Thus, *BC* is 6.

8. $CB = 4x - 9$, $BD = 3x + 5$, and $CD = 17$

SOLUTION:

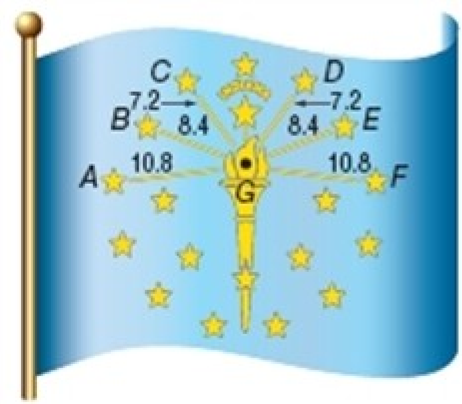
Here *B* is between *C* and *D*.



$$
\begin{aligned}
CD &= CB + BD && \text{Between of points}\\
17 &= 4x - 9 + 3x + 5 && \text{Substitution}\\
17 &= 7x - 4 && \text{Simplify.}\\
17 + 4 &= 7x - 4 + 4 && \text{+4 to each side.}\\
21 &= 7x && \text{Simplify.}\\
\frac{21}{7} &= \frac{7x}{7} && \div \text{ each side by 7.}\\
3 &= x && \text{Simplify.}
\end{aligned}
$$

Then $x = 3$.
Use the value of *x* to find *BC*.
$BC = CB = 4(3) - 9 = 3$

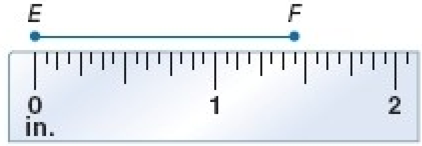
9. **CCSS STRUCTURE** The Indiana State Flag was adopted in 1917. The measures of the segments between the stars and the flame are shown on the diagram in inches. List all of the congruent segments in the figure.



SOLUTION:

Segments that have the same measure are called congruent segments.
Here, $CG = DG = 7.2$, $BG = EG = 8.4$, and $AG = FG = 10.8$.
So, $\overline{CG} \cong \overline{DG}, \overline{BG} \cong \overline{EG}$, and $\overline{AG} \cong \overline{FG}$.

Find the length of each line segment.

10. 

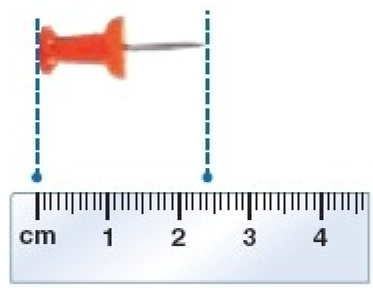
SOLUTION:

The ruler is marked in inches. The distance between two consecutive numbers is divided into 16 equal parts. The starting point *E* of the line segment is at the zero mark of the ruler and the other endpoint *F* is at 7 points after 1. So, the length of the line segment is $1\frac{7}{16}$ in.

11. 

SOLUTION:

The ruler is marked in millimeters. The starting point *X* of the line segment is at the zero mark of the ruler and the other endpoint *Y* is at 8 points after 3. So, the length of the line segment is 3.8 mm.

12. 

SOLUTION:

The ruler is marked in centimeters. One end of the needle stand starts at the zero mark of the ruler and the needle ends at 4 points after 2. So, the length of the needle with the stand is 2.4 cm or 24 mm.

13. Refer to Page 19.

SOLUTION:

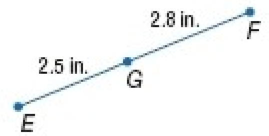
The ruler is marked in inches. The distance between two consecutive numbers is divided into 16 equal parts. One end of the coin starts at the zero mark of the ruler and the other end ends at 15 points after 0.

So, the width of the coin is $\frac{15}{16}$ in.

1-2 Linear Measure

Find the measurement of each segment. Assume that each figure is not drawn to scale.

14. $\overline{EF}$



SOLUTION:

$EF = EG + GF$ Betweenness of points

$EF = 2.5 + 2.8$ Substitution

$EF = 5.3$ Simplify.

So, EF = 5.3 in.

15. $\overline{JL}$

SOLUTION:

$JL = JK + KL$ Betweenness of points

$JL = 0.75 + 0.35$ Substitution

$JL = 1.1$ Addition.

So, JL = 1.1 cm.

16. $\overline{PR}$

SOLUTION:

$PS = PR + RS$ Betweenness of points

$PS - RS = PR + RS - RS$ $-RS$ from each side.

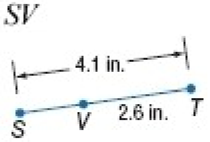
$PS - RS = PR$ Simplify.

$5.8 - 3.7 = PR$ Substitution

$2.1 = PR$ Simplify.

So, PR = 2.1 mm.

17. $\overline{SV}$



SOLUTION:

$ST = SV + VT$ Betweenness of points

$ST - VT = SV + VT - VT$ $-VT$ from each side.

$ST - VT = SV$ Simplify.

$4.1 - 2.6 = SV$ Substitution

$1.5 = SV$ Subtraction.

So, SV = 1.5 in.

18. $\overline{WY}$

SOLUTION:

Segments that have the same measure are called congruent segments.
Here, $WY = YX$. Let $WY = YX = x$.

$WX = WY + YX$ Betweenness of points

$8.8 = x + x$ Substitution

$8.8 = 2x$ Simplify.

$\frac{8.8}{2} = \frac{2x}{2}$ Divide each side by 2

$4.4 = x$ Simplify.

Therefore, WY = 4.4 mm.

19. $\overline{FG}$

SOLUTION:

Segments that have the same measure are called congruent segments.
Here, $\overline{FG} \cong \overline{GH} \cong \overline{HJ} \cong \overline{JK}$.
So, $FG = GH = HJ = JK$. Let each of the lengths be x.

$FK = FG + GH + HJ + JK$ Betweenness of points

$16.8 = x + x + x + x$ Substitution.

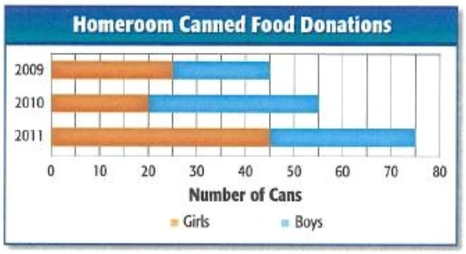
$16.8 = 4x$ Addition.

$\frac{16.8}{4} = \frac{4x}{4}$ Divide each side by 4.

$4.2 = x$ Simplify.

Therefore, FG = 4.2 cm.

20. **CCSS SENSE-MAKING** The stacked bar graph shows the number of canned food items donated by the girls and the boys in a homeroom class over three years. Use the concept of betweenness of points to find the number of cans donated by the boys for each year. Explain your method.



SOLUTION:

The length of the region shaded in orange color represents the number of cans donated by girls and the length shaded in blue represents the number of cans donated by boys.
For 2008, the bar stops at 45 and the orange colored region stops at 25. So, the number of cans donated by girls is 25 and that by boys is 45 – 25 = 20.
Similarly, for 2009, the bar stops at 55 and the orange colored region stops at 20. So, the number of cans donated by girls is 20 and that by boys is 55 – 20 = 35.
For 2010, the bar stops at 75 and the orange colored region stops at 45. So, the number of cans donated by girls is 45 and that by boys is 75 – 45 = 30.

Find the value of the variable and *YZ* if *Y* is between *X* and *Z*.

21. $XY = 11$, $YZ = 4c$, $XZ = 83$

SOLUTION:

Here Y is between X and Z.

So, $XZ = XY + YZ$.

$$\begin{aligned} XZ &= XY + YZ && \text{Betweenness of points} \\ 83 &= 11 + 4c && \text{Substitution} \\ 83 - 11 &= 11 - 11 + 4c && -11 \text{ from each side.} \\ 72 &= 4c && \text{Simplify} \\ \frac{72}{4} &= \frac{4c}{4} && \div \text{ each side by 4.} \\ 18 &= c && \text{Simplify.} \end{aligned}$$

So, $YZ = 4c = 4(18) = 72$.

22. $XY = 6b$, $YZ = 8b$, $XZ = 175$

SOLUTION:

Here Y is between X and Z.

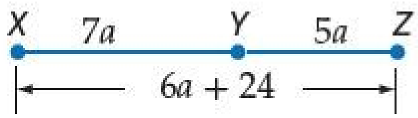
$$\begin{aligned} XZ &= XY + YZ && \text{Betweenness of points} \\ 175 &= 6b + 8b && \text{Substitution} \\ 175 &= 14b && \text{Simplify.} \\ \frac{175}{14} &= \frac{14b}{14} && \text{Divide ech side by 14.} \\ 12.5 &= b && \text{Simplify.} \end{aligned}$$

So, $YZ = 8b = 8(12.5) = 100$.

23. $XY = 7a$, $YZ = 5a$, $XZ = 6a + 24$

SOLUTION:

Here Y is between X and Z.



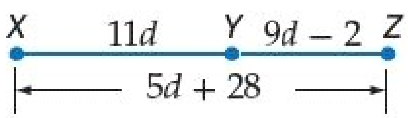
$XZ = XY + YZ$	Betweenness of points
$6a + 24 = 7a + 5a$	Substitution.
$6a + 24 = 12a$	Simplify.
$6a - 6a + 24 = 12a - 6a$	–6a from each side by 14.
$24 = 6a$	Simplify.
$\frac{24}{6} = \frac{6a}{6}$	÷ each side by 6.
$4 = a$	Simplify.

So, $YZ = 5a = 5(4) = 20$.

24. $XY = 11d$, $YZ = 9d - 2$, $XZ = 5d + 28$

SOLUTION:

Y is between X and Z.



$XZ = XY + YZ$	Betweenness of points
$5d + 28 = 11d + 9d - 2$	Substitution
$5d + 28 = 20d - 2$	Simplify.
$5d - 5d + 28 = 20d - 5d - 2$	$-5d$ from each side by 14.
$28 = 15d - 2$	Simplify.
$28 + 2 = 15d - 2 + 2$	$+2$ to each side.
$30 = 15d$	Simplify.
$\frac{30}{15} = \frac{15d}{15}$	÷ each side by 15.
$2 = d$	Simplify.

So, $YZ = 9d = 9(2) - 2 = 16$.

25. $XY = 4n + 3$, $YZ = 2n - 7$, $XZ = 22$

SOLUTION:

Here Y is between X and Z.

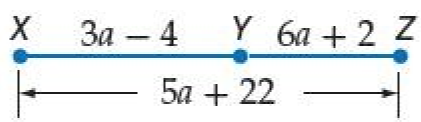
$XZ = XY + YZ$	Betweenness of points
$22 = 4n + 3 + 2n - 7$	Substitution
$22 = 6n - 4$	Simplify.
$22 + 4 = 6n - 4 + 4$	Add 4 to each side.
$26 = 6n$	Simplify.
$\frac{26}{6} = \frac{6n}{6}$	Divide each side by 6.
$\frac{26}{6} \text{ or } 4\frac{1}{3} = n$	Simplify.

$$\text{So, } YZ = 2\left(\frac{13}{3}\right) - 7$$
$$= \frac{26 - 21}{3}$$
$$= \frac{5}{3}$$
$$= 1\frac{2}{3}.$$

26. $XY = 3a - 4$, $YZ = 6a + 2$, $XZ = 5a + 22$

SOLUTION:

Here Y is between X and Z.

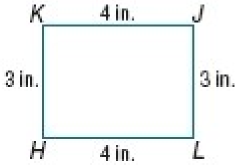


$XZ = XY + YZ$	Betweenness of points
$5a + 22 = 3a - 4 + 6a + 2$	Substitution
$5a + 22 = 9a - 2$	Simplify.
$5a + 2 + 22 = 9a - 2 + 2$	$+2$ to each side.
$5a + 24 = 9a$	Simplify.
$5a - 5a + 24 = 9a - 5a$	$-5a$ from each side.
$24 = 4a$	Simplify.
$\frac{24}{4} = \frac{4a}{4}$	÷ each side by 4.
$6 = a$	Simplify.

So, $YZ = 6a = 6(6) + 2 = 38$.

Determine whether each pair of segments is congruent.

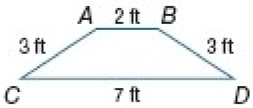
27. $\overline{KJ}, \overline{HL}$



SOLUTION:

Segments that have the same measure are called congruent segments.
Here, $KJ = HL = 4$ in.
Therefore, $\overline{KJ} \cong \overline{HL}$.

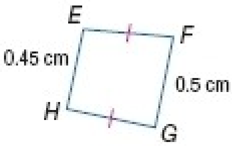
28. $\overline{AC}, \overline{BD}$



SOLUTION:

Segments that have the same measure are called congruent segments.
Here, $AC = BD = 3$ ft.
Therefore, $\overline{AC} \cong \overline{BD}$.

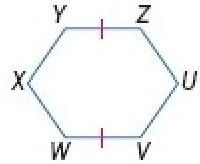
29. $\overline{EH}, \overline{FG}$



SOLUTION:

Segments that have the same measure are called congruent segments.
Here, $EH = 0.45$ cm and $FG = 0.5$ cm. So, $EH \neq FG$. Therefore, $\overline{EH}$ and $\overline{FG}$ are not congruent.

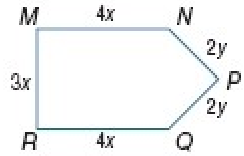
30. $\overline{VW}, \overline{UZ}$



SOLUTION:

Segments that have the same measure are called congruent segments.
Here, the lengths of the segments ZY and VW are given to be equal. But the length of UZ is not known. So, the congruency cannot be determined from the information given.

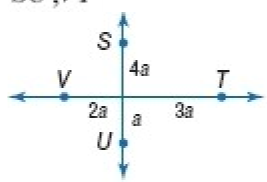
31. $\overline{MN}, \overline{RQ}$



SOLUTION:

Segments that have the same measure are called congruent segments.
Here, $MN = RQ = 4x$.
All segments must have a measure greater than 0.
Therefore, for all $x > 0$, $\overline{MN} \cong \overline{RQ}$.

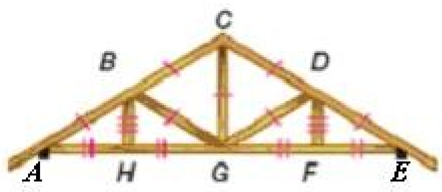
32. $\overline{SU}, \overline{VT}$



SOLUTION:

Segments that have the same measure are called congruent segments.
Here, $SU = 4a + a = 5a$ and $RQ = 2a + 3a = 5a$.
All segments must have a length greater than 0.
Therefore, for all $a > 0$, $\overline{SU} \cong \overline{VT}$.

33. **TRUSSES** A truss is a structure used to support a load over a span, such as a bridge or the roof of a house. List all of the congruent segments in the figure.



SOLUTION:

Segments that have the same measure are called congruent segments.
Here the segments marked with the same symbol are congruent to each other.

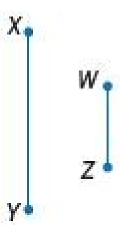
$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{CE} \cong \overline{DG} \cong \overline{BG} \cong \overline{CG}$,

$\overline{AH} \cong \overline{HG} \cong \overline{GF} \cong \overline{FE}$, $\overline{BH} \cong \overline{DF}$,

$\overline{AC} \cong \overline{EC}$, $\overline{AG} \cong \overline{HF} \cong \overline{GE}$

34. **CONSTRUCTION** For each expression:

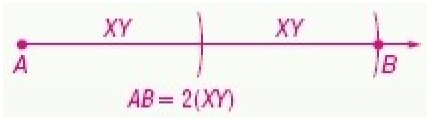
- construct a segment with the given measure,
- explain the process you used to construct the segment, and
- verify that the segment you constructed has the given measure.



a. 2(XY)
b. 6(WZ) – XY

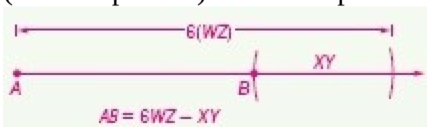
SOLUTION:

a. Sample answer:
Step 1: Draw a line and a point on the line. Label the point *A*.
Step 2: Place the compass at point *X* and adjust the compass setting so that the pencil is at point *Y*.
Step 3: Using that setting, place the compass point at *A* and draw an arc that intersects the line. From that point of intersection, draw another arc in the same direction that intersects the line. Label the second point of intersection *B*.



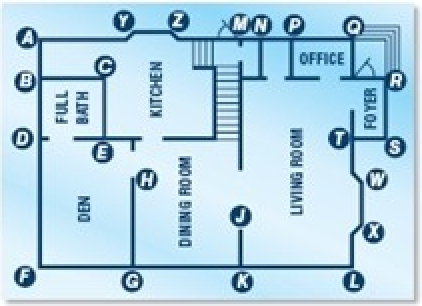
Since same arc measure was used to construct $\overline{XY}$ two times, $AB = 2(XY)$.

b. Sample answer:
Step 1: Draw a line and a point on the line. Label the point *A*.
Step 2: Place the compass at point *W* and adjust the compass setting so that the pencil is at point *Z*.
Step 3: Using the setting, place the compass at *A* and draw an arc the intersects the line. From that point of intersection, draw another arc in the same direction that intersects the line. Continue this until you have marked a total of 6 arcs out from *A*.
Step 4: Place the compass at point *X* and adjust the compass setting so that the pencil is at point *Y*.
Step 5: Using the setting, place the compass at the intersection of the last arc drawn and the line. From that point draw an arc in the opposite direction (towards point *A*). Label the point of intersection *B*.



Since arc measure *WZ* was used six times in the same direction and arc measure *XY* was used to come back in the opposite direction, $AB = 6(WZ) – XY$.

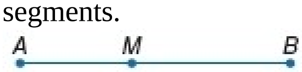
35. **BLUEPRINTS** Use a ruler to determine at least five pairs of congruent segments with labeled endpoints in the blueprint below.



SOLUTION:

When using the student edition, the lengths, $BD = CE = PQ$, $YZ = JK$, $PQ = RS$, and $GK = KL$. Therefore, $\overline{BD} \cong \overline{CE}$; $\overline{BD} \cong \overline{PQ}$; $\overline{YZ} \cong \overline{JK}$; $\overline{PQ} \cong \overline{RS}$; $\overline{GK} \cong \overline{KL}$. For other forms of media, the answer will vary.
Note that you can find congruent segments other than the given too.

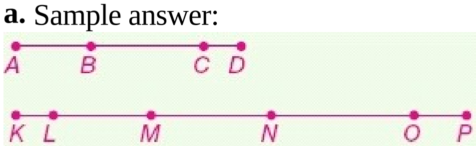
36. **MULTIPLE REPRESENTATIONS** Betweenness of points ensures that a line segment may be divided into an infinite number of line segments.



a. GEOMETRIC Use a ruler to draw a line segment 3 centimeters long. Label the endpoints A and D. Draw two more points along the segment and label them B and C. Draw a second line segment 6 centimeters long. Label the endpoints K and P. Add four more points along the line and label them L, M, N, and O.
b. TABULAR Use a ruler to measure the length of the line segment between each of the points you have drawn. Organize the lengths of the segments in $\overline{AD}$ and $\overline{KP}$ into a table. Include a column in your table to record the sum of these measures.
c. ALGEBRAIC Give an equation that could be used to find the lengths of $\overline{AD}$ and $\overline{KP}$. Compare the lengths determined by your equation to the actual lengths.

SOLUTION:

a. Sample answer:



b. Sample answer:

$\overline{AD}$	
Segment	Length (cm)
$\overline{AB}$	1.0
$\overline{BC}$	1.5
$\overline{CD}$	0.5
Total	3.0

$\overline{KP}$	
Segment	Length (cm)
$\overline{KL}$	0.5
$\overline{LM}$	1.3
$\overline{MN}$	1.6
$\overline{NO}$	1.9
$\overline{OP}$	0.7
Total	6.0

c.

$AD = AB + BC + CD$; $KP = KL + LM + MN + NO + OP$

$AB + BC + CD = 3$; $KL + LM + MN + NO + OP = 6$

The lengths of each segment add up to the length of the whole segment.

37. **WRITING IN MATH** If point B is between points A and C, explain how you can find AC if you know AB and BC. Explain how you can find BC if you know AB and AC.

SOLUTION:

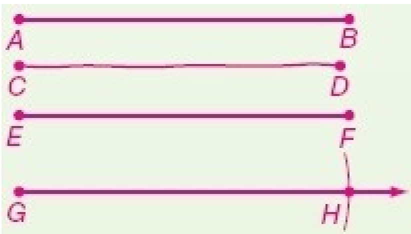
If point B is between points A and C then, $AC = AB + BC$. So, if you know AB and BC, add AB and BC to find AC.
If you know AB and AC, solve the equation to find BC.

$$\begin{aligned} AC &= AB + BC \\ AC - AB &= AB + BC - AB \\ AC - AB &= BC \end{aligned}$$

Therefore, subtract AB from AC to find BC.

38. **OPEN ENDED** Draw a segment $\overline{AB}$ that measures between 2 and 3 inches long. Then sketch a segment $\overline{CD}$ congruent to $\overline{AB}$, draw a segment $\overline{EF}$ congruent to $\overline{AB}$, and construct a segment $\overline{GH}$ congruent to $\overline{AB}$. Compare your methods.

SOLUTION:

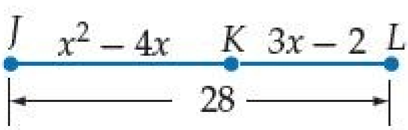


Both $\overline{AB}$ and $\overline{EF}$ were created using a ruler, while $\overline{GH}$ was created using a straightedge and compass and $\overline{CD}$ was created without any of these tools. $\overline{AB}$, $\overline{EF}$, and $\overline{GH}$ have the same measure, but $\overline{CD}$ not only does not have the same length, it isn't even a straight line.

39. **CHALLENGE** Point *K* is between points *J* and *L*. If $JK = x^2 - 4x$, $KL = 3x - 2$, and $JL = 28$, write and solve an equation to find the lengths of *JK* and *KL*.

SOLUTION:

Here *K* is between *J* and *L*.



$JL = JK + KL$	Betweenness of points
$28 = x^2 - 4x + 3x - 2$	Substitution
$28 = x^2 - x - 2$	Simplify.
$28 - 28 = x^2 - x - 2 - 28$	−28 from each side.
$0 = x^2 - x - 30$	Simplify.
$0 = (x - 6)(x + 5)$	Factor.
$x = 6 \text{ or } -5$	Simplify.

Since *x* is a length, it cannot be negative. So, $x = 6$. Use the value of *x* to find *JK* and *KL*.

$JK = x^2 - 4x = (6)^2 - 4(6) = 12$
$KL = 3x - 2 = 3(6) - 2 = 16$

40. **CCSS REASONING** Determine whether the statement *If point M is between points C and D, then CD is greater than either CM or MD is sometimes, never,* or *always* true. Explain.

SOLUTION:

The statement is always true. If the point *M* is between the points *C* and *D*, then $CM + MD = CD$. Since measures cannot be negative, *CD*, which represents the whole, must always be greater than either of the lengths of its parts, *CM* or *MD*.

41. **WRITING IN MATH** Why is it important to have a standard of measure?

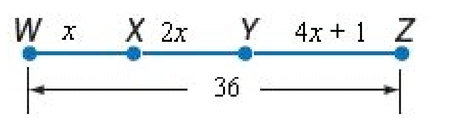
SOLUTION:

Sample answer: Units of measure are used to differentiate between size and distance, as well as for precision. An advantage is that the standard of measure of a cubit is always available. A cubit was the length of an arm from the elbow to the finger tips. A disadvantage is that a cubit would vary in length depending on whose arm was measured.

42. **SHORT RESPONSE** A 36-foot-long ribbon is cut into three pieces. The first piece of ribbon is half as long as the second piece of ribbon. The third piece is 1 foot longer than twice the length of the second piece of ribbon. How long is the longest piece of ribbon?

SOLUTION:

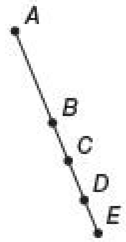
Let *x* be the length of the first piece. Then the second piece is 2*x* feet long and the third piece is $2(2x) + 1 = 4x + 1$ feet long. The total length of the ribbon is 36 feet.



$WZ = WX + XY + YZ$	Betweenness of points
$36 = x + 2x + 4x + 1$	Substitution
$36 = 7x + 1$	Simplify.
$36 - 1 = 7x + 1 - 1$	−1 from each side.
$35 = 7x$	Simplify.
$\frac{35}{7} = \frac{7x}{7}$	÷ each side by 7.
$5 = x$	Simplify.

So, the length of the longest piece is $4x + 1 = 4(5) + 1 = 21$ feet.

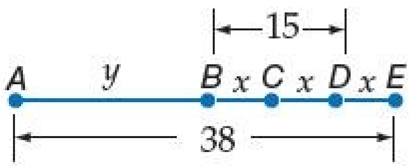
43. In the figure, points A, B, C, D, and E are collinear. If $AE = 38$, $BD = 15$, and $\overline{BC} \cong \overline{CD} \cong \overline{DE}$, what is the length of $\overline{AD}$?



A 7.5
B 15
C 22.5
D 30.5

SOLUTION:

Let $BC = CD = DE = x$ and $AB = y$.



$BD = BC + CD$	Betweenness of points
$15 = x + x$	Substitution
$15 = 2x$	Simplify.
$\frac{15}{2} = \frac{2x}{2}$	Divide each side by 2.
$7.5 = x$	Simplify.

$AE = AB + BC + CD + DE$	Betweenness of points
$38 = y + 3x$	Substitution
$38 = y + 3(7.5)$	Substitution
$38 = y + 22.5$	Simplify.
$38 - 22.5 = y + 22.5 - 22.5$	− 22.5 from each side.
$15.5 = y$	Simplify.

$AB = y = 15.5$
Then, $AD = AB + BD = 15.5 + 15 = 30.5$.
Therefore, the correct choice is D.

44. **SAT/ACT** If $f(x) = 7x^2 - 4x$, what is the value of $f(2)$?
F -8
G 2
H 6
J 17
K 20

SOLUTION:

Substitute $x = 2$ in the expression $7x^2 - 4x$.

$f(x) = 7x^2 - 4x$	Original function.
$f(2) = 7(2)^2 - 4(2)$	Replace x with 2.
$= 28 - 8$	Simplify.
$= 20$	Simplify.

Therefore, the correct choice is K.

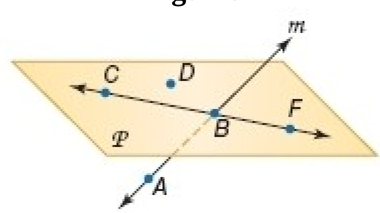
45. **ALGEBRA**
Simplify $(3x^2 - 2)(2x + 4) - 2x^2 + 6x + 7$.
A $4x^2 + 14x - 1$
B $4x^2 - 14x + 15$
C $6x^3 + 12x^2 + 2x - 1$
D $6x^3 + 10x^2 + 2x - 1$

SOLUTION:

$(3x^2 - 2)(2x + 4) - 2z^2 + 6x + 7$
$= (6x^3 + 12x^2 - 4x - 8) - 2z^2 + 6x + 7$ Multiply binomials.
$= 6x^3 + 10x^2 + 2x - 1$ Simplify.

Therefore, the correct choice is D.

Refer to the figure.



46. What are two other names for $\overleftrightarrow{AB}$?

SOLUTION:

A line extends in both the directions, so it can be named using the points in any order. So, the line $\overleftrightarrow{AB}$ is same as $\overleftrightarrow{BA}$.
The line is also labeled *m*.

47. Give another name for plane *P*.

SOLUTION:
A plane is determined by three points. Here, the plane *P* is the same as the plane *CDF*.

48. Name the intersection of plane *P* and $\overleftrightarrow{AB}$.

SOLUTION:
The line $\overleftrightarrow{AB}$ meets the plane *P* at *B*. So, the point of intersection is *B*.

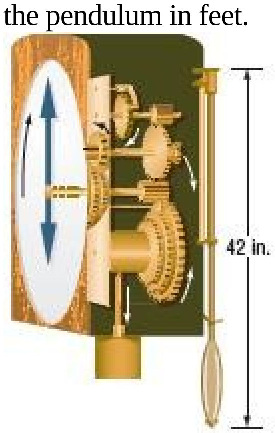
49. Name three collinear points.

SOLUTION:
Collinear points are points that lie on the same line. Here, the points *C*, *B*, and *F* lies on the same line $\overleftrightarrow{CF}$. Therefore, they are collinear.

50. Name two points that are not coplanar.

SOLUTION:
Coplanar points are points that lie in the same plane. Here, the point *A* is not on the plane *P*. So, any point on the plane *P* is non coplanar to *A*. The points *A* and *D* are not coplanar.

51. **CLOCKS** The period of a pendulum is the time required for it to make one complete swing back and forth. The formula of the period *P* in seconds of a pendulum is $P = 2\pi\sqrt{\frac{\ell}{32}}$, where ℓ is the length of the pendulum in feet.



a. What is the period of the pendulum in the clock shown to the nearest tenth of a second?
b. About how many inches long should the pendulum be in order for it to have a period of 1 second?

SOLUTION:
a. First convert the length of the pendulum into feet.

One foot is equivalent to 12 inches. So, 42 inches is equivalent to 3.5 feet.
Substitute ℓ = 3.5 in the equation.

$$P = 2\pi\sqrt{\frac{\ell}{32}} \quad \text{Original equation}$$
$$P = 2\pi\sqrt{\frac{3.5}{32}} \quad \text{Replace } \ell \text{ with } 3.5.$$
$$= 2.0788.. \quad \text{Simplify.}$$
$$\approx 2.1$$

Therefore, the period of the pendulum is about 2.1 seconds.

b.
$$P = 2\pi\sqrt{\frac{\ell}{32}} \quad \text{Original equation}$$
$$1 = 2\pi\sqrt{\frac{\ell}{32}} \quad \text{Replace } P \text{ with 1.}$$
$$(1)^2 = \left(2\pi\sqrt{\frac{\ell}{32}}\right)^2 \quad \text{Square both sides.}$$
$$1 = \frac{4\pi^2\ell}{32} \quad \text{Simplify.}$$
$$1 \cdot \frac{32}{4\pi^2} = \frac{4\pi^2\ell}{32} \cdot \frac{32}{4\pi^2} \quad \text{Mutliply each side by } \frac{32}{4\pi^2}.$$
$$\ell = \frac{32}{4\pi^2} \quad \text{Simplify.}$$
$$= 0.809917... \quad \text{Simplify.}$$
$$\approx 0.81$$

The pendulum should be about 0.81 feet, that is, about 0.81(12) = 9.72 inches long.

Solve each inequality.

52. $-14n \geq 42$

SOLUTION:
$$\frac{-14n}{-14} \geq \frac{42}{-14} \quad \text{Original inequality}$$
$$\frac{-14n}{-14} \leq \frac{42}{-14} \quad \text{Divide each side by } -14.$$
$$n \leq -3 \quad \text{Simplify.}$$

If each side of an inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be reversed so that the resulting inequality is true.
Therefore, the solution set for the inequality is $\{n \mid n \leq -3\}$.

53. $p+6>15$

SOLUTION:

$$\begin{aligned} p+6&>15 && \text{Original inequality}\\ p+6-6&>15-6 && \text{Subtract 6 from each side.}\\ p&>9 && \text{Simplify.}\end{aligned}$$

Therefore, the solution set for the inequality is $\{p \mid p>9\}$.

54. $-2a-5<20$

SOLUTION:

$$\begin{aligned} -2a-5&<20 && \text{Original inequality}\\ -2a-5+5&<20+5 && \text{Add 5 to each side}\\ -2a&<25 && \text{Simplify.}\\ \frac{-2a}{-2}&>\frac{25}{-2} && \text{Divide each side by } -2.\\ a&>-12.5 && \text{Simplify.}\end{aligned}$$

If each side of an inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be reversed so that the resulting inequality is true.

Therefore, the solution set for the inequality is $\{a \mid a>-12.5\}$.

55. $5x \le 3x-26$

SOLUTION:

$$\begin{aligned} 5x&\le 3x-26 && \text{Original inequality}\\ 5x-3x&\le 3x-3x-26 && -3x \text{ from each side}\\ 2x&\le -26 && \text{Simplify.}\\ \frac{2x}{2}&\le\frac{-26}{2} && \div \text{ each side by 2}\\ x&\le -13 && \text{Simplify.}\end{aligned}$$

Therefore, the solution set for the inequality is $\{x \mid x\le -13\}$.

Evaluate each expression if *a* = –7, *b* = 4, *c* = –3, and *d* = 5.

56. *b* – *c*

SOLUTION:

Substitute *b* = 4 and *c* = –3.

$$\begin{aligned} b-c&=4-(-3) && \text{Substitution.}\\ &=4+3 && \text{Simplify.}\\ &=7 && \text{Addition.}\end{aligned}$$

57. $|a-d|$

SOLUTION:

Substitute *a* = –7 and *d* = 5.

$$\begin{aligned} |a-d|&=|-7-5| && \text{Substitution.}\\ &=|-12| && \text{Subtraction.}\\ &=12 && |-12|=12.\end{aligned}$$

58. $|d-c|$

SOLUTION:

Substitute *d* = 5 and *c* = –3.

$$\begin{aligned} |d-c|&=|5-(-3)| && \text{Substitution}\\ &=|5+3| && \text{Simplify.}\\ &=|8| && \text{Addition.}\\ &=8 && |8|=8.\end{aligned}$$

59. $\frac{b-a}{2}$

SOLUTION:

Substitute *b* = 4 and *a* = –7.

$$\begin{aligned} \frac{b-a}{2}&=\frac{4-(-7)}{2} && \text{Substitution.}\\ &=\frac{4+7}{2} && \text{Simplify.}\\ &=\frac{11}{2} && \text{Addition.}\\ &=5.5 && \text{Division.}\end{aligned}$$

60. $(a-c)^2$

SOLUTION:

Substitute *a* = –7 and *c* = –3.

$$\begin{aligned} (-a-c)^2&=(-7-(-3))^2 && \text{Substitution.}\\ &=(-7+3)^2 && \text{Simplify.}\\ &=(-4)^2 && \text{Addition.}\\ &=16 && \text{Square } -4.\end{aligned}$$

61. $\sqrt{(a-b)^2+(c-d)^2}$

SOLUTION:

Substitute *a* = –7, *b* = 4, *c* = –3, and *d* = 5.

$$\sqrt{(a-b)^2+(c-d)^2}$$

$=\sqrt{(-7-4)^2+(-3-5)^2}$	Substitution.
$=\sqrt{(-11)^2+(-8)^2}$	Subtraction.
$=\sqrt{121+64}$	Square terms.
$=\sqrt{185}$	Addition.